CBL 4-4

Name ______________________ Class ______________ Date ____________

CBL Lab B

Air Resistance

Planning a Wilderness Supply Drop

You are part of a technical support crew that will be delivering delicate weather equipment to scientists in remote wilderness areas of Alaska. The equipment will be delivered by plane, using supply boxes attached to parachutes. The boxes are designed to withstand impact at velocities up to 1.5 m/s. The heaviest piece of equipment you must deliver weighs about 55 kg.

You have been asked to recommend an appropriate parachute (one that offers adequate air resistance) and to determine a maximum possible load per drop. To do so, you must first determine the mathematical model that best describes the relationship between the mass of an object, the air resistance on the object, and the terminal speed the object reaches.

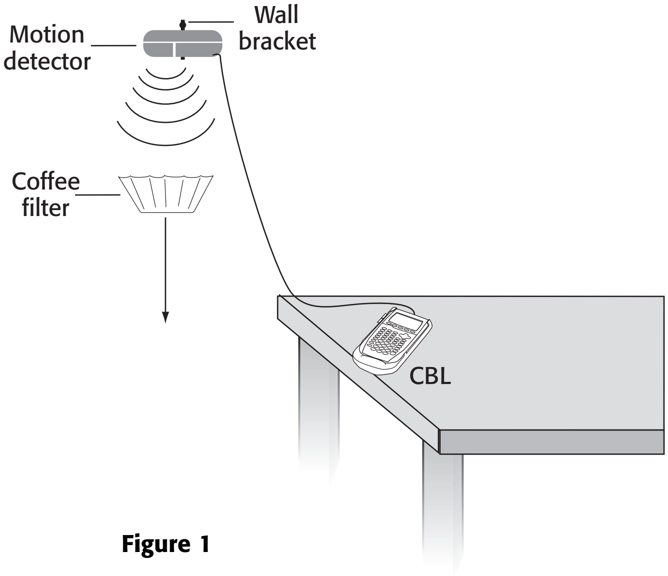

Figure 1

OBJECTIVES

Develop a physical model that simulates an object falling with a parachute.

Analyze the relationship between mass and terminal speed of a falling object using data from the model.

Evaluate two mathematical models relating air resistance to terminal speed using data generated in the simulation.

Calculate an air resistance factor for the model.

Predict the required air resistance factor for the parachute and the maximum allowable load for the drop.

MATERIALS LIST

- graphing calculator with link cable
- CBL 2™ or LabPro® system
- DataMate® application loaded in calculator
- Vernier motion detector
- coffee filters, (5 basket-style)
- balance
- graph paper

SAFETY

- Perform this experiment in a clear area. Attach masses securely. Falling, dropped, or swinging masses can cause serious injury. Use caution when standing on ladders or chairs.
- Tie back long hair, secure loose clothing, and remove loose jewelry to keep them from getting caught in moving or rotating parts.

Developing the Model

To determine a mathematical model for air resistance in the laboratory, you can drop an object that is similar to a parachute and use a motion detector to collect data as it falls. To simulate a parachute in this experiment, you will use coffee filters dropped right-side-up. The factors affecting the upward force of air resistance on the filters will be combined into a single number called the "air resistance factor."

Answer the following questions before starting this activity.

1. For a falling object, what is the mathematical expression that describes the downward force acting on the object?

2. When an object reaches terminal speed, what is the net force acting on it? Explain your answer.

CBL 4-4

3. Sketch a graph of velocity versus time for a parachute carrying a small load and that is falling through the air. Label the point on the graph at which terminal speed occurs. On the same set of axes, sketch a graph of a parachute with a much larger weight attached. How does the weight of the load affect the terminal speed the parachute reaches?

4. One possible mathematical model for the air resistance on a parachute is that the air resistance is directly proportional to the speed ($F_R = -kv$). Assuming this is the case, find an expression for terminal speed, v_T, in terms of g, m, and k, where g is the free-fall acceleration, m is mass, and k is a constant air resistance factor. (Hint: Set $F_R = -F_g$ and solve for v.)

5. Another possible mathematical model for the air resistance on a parachute is that the air resistance is directly proportional to the square of the speed ($F_R = -kv^2$). Assuming this is the case, find an expression for terminal speed, v_T, in terms of g, m, and k.

Procedure

1. Mount the motion detector at least two meters above the ground using a bracket on the wall or ceiling. Orient the motion detector so that it faces straight down toward the floor.
2. Connect the motion detector to the DIG/SONIC port of the CBL 2™ or LabPro® unit. Use the black link cable to connect the CBL unit to the calculator. Firmly press in the cable ends.
3. Turn on the calculator and start the DataMate® application. Press CLEAR to reset the application.
4. Determine the mass of a single coffer filter and record the mass in a data table like the one shown in the Data Table section.
5. Make sure the area around you is free of obstructions. Hold the coffee filter about 0.5 m under the motion detector, as shown in **Figure 1.** Do not hold the filter closer than 0.4 m. Select START to begin data collection.
6. When the motion detector begins to click, release the coffee filter directly below the motion detector so that the coffee filter falls toward the floor. Move your hands out of the beam of the motion detector as quickly as possible so that only the motion of the filter is recorded.
7. View a graph of distance versus time.
   - Press ENTER to view the distance-time graph.
   - If the motion of the filter was too erratic to get a smooth graph, repeat the measurement. With practice, you should be able to release the filter so that it falls almost straight down with little sideways motion.
   - Press ENTER, and select MAIN SCREEN.
   - Repeat data collection as necessary.
8. The speed of the coffee filter can be determined from the slope of the distance-time graph. At the start of the graph, there should be a region of increasing slope (increasing speed), and then the plot should become linear because the filter was falling with a constant or terminal speed (v_T) during that time. To fit a line to only the linear region:
   - Select GRAPH from the main screen.
   - To select just the linear portion of the distance graph, select SELECT REGION.
   - Use the left and right arrow keys to move the flashing cursor to the left edge of the linear region corresponding to the filter in motion at constant speed, and press ENTER.
   - Move the flashing cursor to the right edge of the linear region, and press ENTER.

- View your abbreviated graph by pressing ENTER. You should see only the linear region.
- Select MAIN SCREEN to return to the main screen.
- Select ANALYZE from the main screen.
- Select CURVE FIT from ANALYZE OPTIONS.
- Select LINEAR (DIST VS TIME) from the CURVE FIT screen.
- Record the slope in the data table (a velocity in m/s).
- Press ENTER to see the fit along with your data.
- Press ENTER, and select RETURN TO MAIN SCREEN.

9. Repeat steps 4–8 for nested stacks of two, three, four, and five coffee filters. (Optionally extend to six, seven, and eight filters, but be sure to allow sufficient falling distance so that a terminal speed will be reached.) Record all data in your data table.

DATA TABLE

Number of filters	Total mass, m (kg)	Terminal speed, v_T (m/s)	v_T^2 (m^2/s^2)	Drag coefficient (kg/m)
1				
2				
3				
4				
5				
			Average =	

Analysis

1. Graphing Data Use your calculator, the graph at right, or graph paper to plot terminal speed, v_T, versus mass, m, for the five trials. Be sure to scale the axes from the origin (0,0). Draw a line through your data that also goes through the origin. Does your data fit a linear model? Explain why or why not.

2. **Calculate** Square each terminal speed in the data table, and record the results under the heading v_T^2 (m^2/s^2) in your data table.

3. **Graphing Data** On a separate graph, plot terminal speed squared, v_T^2, versus mass, m. Again, scale the axes through the origin. Does this seem to be a better fit than the linear model? Explain why or why not.

4. **Interpreting Graphs** Based on your data and graphs, which mathematical model best represents the relationship between the force of air resistance and the speed of the coffee filters? (Choose a or b.)

   a. $F_R = -kv$ (linear model)

   b. $F_R = -kv^2$ (quadratic model)

5. **Evaluating Results** Calculate an air resistance factor, k, for each of the coffee filter trials. If you found that your data fit a linear model better, use the following equation:

$$k = \frac{mg}{v_T}$$

If you found that your data fit a quadratic model best, use the following equation instead:

$$k = \frac{mg}{{v_T}^2}$$

Record the values for k in your data table, then calculate an average air resistance factor for all the trials combined.

Conclusions

6. **Applying the Model** Assume that air resistance on the parachute follows the same mathematical model as the coffee filters. If the support team decided to deliver equipment in batches weighing 200 kg, what would the minimum required air resistance factor for the parachute be? Remember that the impact speed can be no greater than 1.5 m/s. (Hint: Use one of the equations from step 4 in the Analysis.)

7. **Making Predictions** Your current parachutes have air resistance factors of 160 kg/m. What is the maximum load you can deliver without exceeding the impact speed of 1.5 m/s? The heaviest single piece of equipment that must be delivered weighs 55 kg. Will your team be able to use the current parachute to safely deliver this equipment? (Hint: Use one of the equations from step 5 in the Analysis section and solve for mass, m.)

8. **Applying the Model** On a recent drop, one of the parachutes did not open. The supply box, with a mass of 200 kg and an air resistance factor of 0.8, continued to accelerate without its parachute. What was the terminal speed that was reached by the supply box?

9. **Evaluating Models** If a parachute and a coffee filter each had the same cross-sectional area, which would offer more air resistance? Explain why you think so. Do you think the differences between them would result in an entirely different mathematical model?

____

____

____

____

____

Extensions

1. **Evaluating Models** Design a small parachute and have your teacher approve your design. Use the motion detector to analyze the air resistance and terminal speed as the weight suspended from the parachute increases. Determine whether or not the parachute uses the same mathematical model as the coffee filter.
2. **Developing Models** The air resistance factor, k, used in this experiment combines several factors into a single constant. When physicists study air resistance, they sometimes use the following equation to summarize the force of air resistance:

$$F_R = (1/2)\bullet C_D\bullet A\bullet r\bullet v^2$$

where C_D is a *drag coefficient* based on the overall shape of an object (a perfect sphere has a $C_D = 0.5$), A is the cross-sectional area in m^2 of the falling object, and r refers to the characteristics of the medium through which the object is falling (in air, r is equal to 1.2 kg/m^3). Use this equation and your data to calculate the drag coefficient, C_D, for the coffee filter.

3. **Applying the Model** Look up the drag coefficients for several standard parachute designs, and calculate the size parachute required with each design to safely deliver a 55 kg load with a terminal velocity no greater than 1.5 m/s. Use the equation in item 2 above.
4. **Applying the Model** A 65 kg stuntwoman jumps from a plane using a giant coffee filter as a parachute. Use the drag coefficient, C_D, that you calculated in item 2 above to determine the necessary cross-sectional area of the giant filter if she does not want to exceed a speed of 3 m/s while falling.