Name _____ Class _____ Date _____

Skills Worksheet

Problem Solving

Significant Figures

A lever balance used to weigh a truckload of stone may be accurate to the nearest 100 kg, giving a reading of 15 200 kg, for instance. The measurement should be written in such a way that a person looking at it will understand that it represents the mass of the truck to the nearest 100 kg, that is, that the mass is somewhere between 15 100 kg and 15 300 kg.

Some laboratory balances are sensitive to differences of 0.001 g. Suppose you use such a balance to weigh 0.206 g of aluminum foil. A person looking at your data table should be able to see that the measurement was made on a balance that measures mass to the nearest 0.001 g. You should not state the measurement from the laboratory balance as 0.2060 g instead of 0.206 g because the balance was not sensitive enough to measure 0.0001 g.

To convey the accuracy of measurements, all people working in science use significant figures. *A significant figure is a digit that represents an actual measurement.* The mass of the truck was stated as 15 200 kg. The 1, 5, and 2 are significant figures because the balance was able to measure ten-thousands, thousands, and hundreds of kilograms. The truck balance was not sensitive enough to measure tens of kilograms or single kilograms. Therefore, the two zeros are not significant and the measurement has three significant figures. The mass of the foil was correctly stated as 0.206 g. There are three decimal places in this measurement that are known with some certainty. Therefore, this measurement has three significant figures. Had the mass been stated as 0.2060 g, a fourth significant figure would have been incorrectly implied.

Rules for Determining Significant Figures

A. All digits that are not zeros are significant.

All are nonzero digits.

↓ ↓ ↓

3 2 5 mL of ethanol

The measurement has three significant figures.

All are nonzero digits.

↓ ↓ ↓ ↓

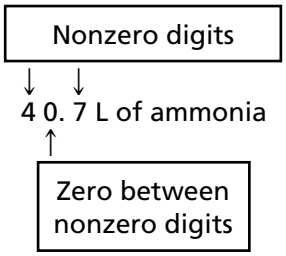
1.3 2 5 g of zinc

The measurement has four significant figures.

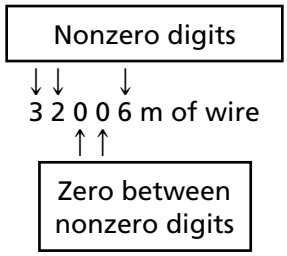
B. Zeros may or may not be significant. To determine whether a zero is significant, use the following rules:

Problem Solving *continued*

1. Zeros appearing between nonzero digits are significant.

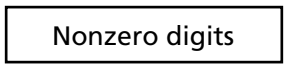


The measurement has three significant figures.

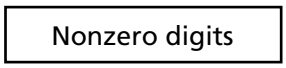


The measurement has five significant figures.

2. Zeros appearing in front of nonzero digits are not significant.

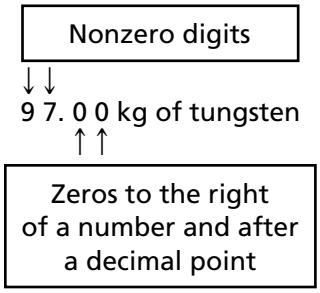


The measurement has three significant figures.

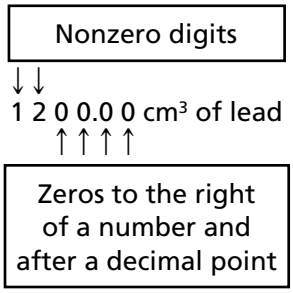


The measurement has one significant figure.

3. Zeros at the end of a number and to the right of a decimal are significant figures. Zeros between nonzero digits and significant zeros are also significant. This is a restatement of Rule 1.



The measurement has four significant figures.



The measurement has six significant figures.

4. Zeros at the end of a number but to the left of a decimal may or may not be significant. If such a zero has been measured or is the first estimated digit, it is significant. On the other hand, if the zero has not been measured or estimated but is just a place holder, it is not significant. A decimal placed after the zeros indicates that they are significant.

Problem Solving *continued*

Nonzero digits

↓ ↓

3 4 0 0 g of sulfur

The measurement has two significant figures.

Nonzero digits

↓ ↓

4 0 0 0. mL of oxygen

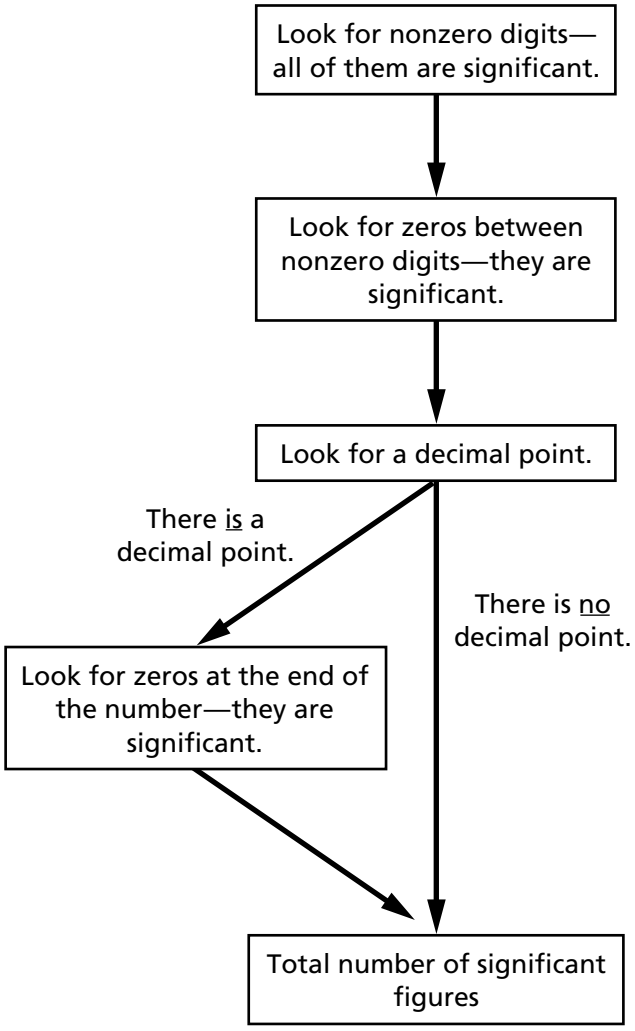
↑ ↑

Decimal point is present, so these zeros are significant.

The measurement has four significant figures.

The rules are summarized in the following flowchart:

General Plan for Determining Significant Figures



Sample Problem 1

Determine the number of significant figures in the following measurements:

a. 30 040 g
b. 0.663 kg
c. 20.05 mL
d. 1500. mg
e. 0.0008 m

Solution

ANALYZE

What is given in the problem? **five measurements**

What are you asked to find? **the number of significant figures in each measurement**

Items	Data				
	a	b	c	d	e
Measured quantity	30 040 g	0.663 kg	20.05 L	1500. mg	0.0008 g

PLAN

What steps are needed to determine the number of significant figures in each measurement?

Apply the steps in the flowchart to determine the number of significant figures.

Apply the following steps from the flowchart. Eliminate the steps that are not applicable to the measurement in question.

How many nonzero digits are there?	?
How many zeros are there between nonzero digits?	?
Is there a decimal point?	?
How many significant zeros are at the end of the number?	?
Total number of significant figures	?

SOLVE

a. 30 040 g

How many nonzero digits are there?	2
How many zeros are there between nonzero digits?	2
Is there a decimal point?	no
How many significant zeros are at the end of the number?	NA
Total number of significant figures	4

The final zero is not significant.

b. 0.663 kg

How many nonzero digits are there?	3
How many zeros are there between nonzero digits?	NA
Is there a decimal point?	yes
How many significant zeros are at the end of the number?	NA
Total number of significant figures	3

The zero only locates the decimal point and is not significant.

c. 20.05 L

How many nonzero digits are there?	2
How many zeros are there between nonzero digits?	2
Is there a decimal point?	yes
How many significant zeros are at the end of the number?	NA
Total number of significant figures	4

d. 1500. mg

How many nonzero digits are there?	2
How many zeros are there between nonzero digits?	NA
Is there a decimal point?	yes
How many significant zeros are at the end of the number?	2
Total number of significant figures	4

There is a decimal following the final two zeros, so all digits are significant.

e. 0.0008 g

How many nonzero digits are there?	1
How many zeros are there between nonzero digits?	NA
Is there a decimal point?	yes
How many significant zeros are at the end of the number?	NA
Total number of significant figures	1

The zeros are only place holders. They are not significant.

EVALUATE

Are the answers reasonable?

Yes; all answers are in agreement with the rules for determining significant figures.

Practice

1. Determine the number of significant figures in the following measurements:

a. 640 cm^3 **ans: 2**

b. 200.0 mL **ans: 4**

c. 0.5200 g **ans: 4**

d. 1.005 kg **ans: 4**

e. 10 000 L **ans: 1**

f. 20.900 cm **ans: 5**

g. 0.000 000 56 g/L **ans: 2**

h. 0.040 02 kg/m^3 **ans: 4**

i. 790 001 cm^2 **ans: 6**

j. 665.000 $kg \cdot m/s^2$ **ans: 6**

DETERMINING SIGNIFICANT FIGURES IN CALCULATIONS

Suppose you want to determine the density of an ethanol-water solution. You first measure the volume in a graduated cylinder that is accurate to the nearest 0.1 mL. You then determine the mass of the solution on a balance that can measure mass to the nearest 0.001 g. You have read each measuring device as accurately as you can, and you record the following data:

Measurement	Data
Mass of solution, m	11.079 g
Volume of solution, V	12.7 mL
Density of solution in g/mL, D	?

You can determine density on your calculator and get the following result:

$$D = \frac{m}{V} = \frac{11.079\ \text{g}}{12.7\ \text{mL}} = 0.872\ 362\ 204\ \text{g/mL}$$

Although the numbers divide out to give the result shown, it is not correct to say that this quantity is the density of the solution. Remember that you are dealing with measurements, not just numbers. Consider the fact that you measured the mass of the solution with a balance that gave a reading with five significant figures: 11.079 g. In addition, you measured the volume of the solution with a graduated cylinder that was readable only to three significant figures: 12.7 mL. It seems odd to claim that you now know the density with an accuracy of nine significant figures.

You can calculate the density—or any measurement—*only as accurately as the least accurate measurement* that was used in the calculation. In this case the least accurate measurement was the volume because the measuring device you used was capable of giving you a measurement with only three significant figures. Therefore, you can state the density to only three significant figures.

Rules for Calculating with Measured Quantities

Operation	Rule
Multiplication and division	• Round off the calculated result to the same number of significant figures as the measurement having the fewest significant figures.
Addition and subtraction	• Round off the calculated result to the same number of decimal places as the measurement with the fewest decimal places. If there is no decimal point, round the result back to the digit that is in the same position as the leftmost uncertain digit in the quantities being added or subtracted.

In the example given above, you must round off your calculator reading to a value that contains three significant figures. In this case, you would say:

$$D = \frac{m}{V} = \frac{11.079\ \text{g}}{12.7\ \text{mL}} = 0.872\ \cancel{362\ 204}\ \text{g/mL} = 0.872\ \text{g/mL}$$

Sample Problem 2

In an experiment to identify an unknown gas, it is found that 1.82 L of the gas has a mass of 5.430 g. What is the density of the gas in g/L?

Solution

ANALYZE

What is given in the problem? **the measured mass and volume of the gas**

What are you asked to find? **the density of the gas**

Items	Data
Mass of the gas, m_{gas}	5.430 g
Volume of the gas, V_{gas}	1.82 L
Density of the gas, D_{gas} (numerical result)	? g/L
Least number of significant figures in measurements	3 (in 1.82 L)
Density of the gas, D_{gas} (rounded)	? g/L

PLAN

What step is needed to calculate the density of the gas?

Divide the mass measurement by the volume measurement.

What steps are necessary to round the calculated value to the correct number of significant figures?

Determine which measurement has the fewest significant figures. Round the calculated result to that number of significant figures.

$$D_{gas} = \frac{m_{gas}}{V_{gas}} = \text{numerical result} \xrightarrow{\text{round to correct significant figures}} \text{rounded result}$$

COMPUTE

$$D_{gas} = \frac{m_{gas}}{V_{gas}} = \frac{\overset{\text{four significant figures}}{5.430\text{ g}}}{\underset{\text{three significant figures}}{1.82\text{ L}}} = \overset{\text{round to three significant figures}}{2.983\,\cancel{516\,484}} = 2.98\text{ g/L}$$

the digit following the 8 is less than 5, so the 8 remains unchanged

EVALUATE

Are the units correct?

Yes; density is given in units of mass per unit volume.

Are the significant figures correct?

Yes; the mass had only three significant figures, so the answer was rounded to three significant figures.

Is the answer reasonable?
Yes; the mass/volume ratio is roughly 3/1, so the density is approximately 3 g/L.

Practice

1. Perform the following calculations, and express the result in the correct units and number of significant figures.

a. 47.0 ÷ 2.2 s **ans: 21 m/s**

b. 140 cm × 35 cm **ans: 4900 cm^2**

c. 5.88 kg ÷ 200 m^3 **ans: 0.03 kg/m^3**

d. 0.00 50 m^2 × 0.042 m **ans: 0.000 21 m^3**

e. 300.3 L ÷ 180. s **ans: 1.67 L/s**

f. 33.00 cm^2 × 2.70 cm **ans: 89.1 cm^3**

g. 35 000 kJ ÷ 0.250 min **ans: 140 000 kJ/min**

Sample Problem 3

Three students measure volumes of water with three different devices. They report the following volumes:

Device	Volume measured
Large graduated cylinder	**164 mL**
Small graduated cylinder	**39.7 mL**
Calibrated buret	**18.16 mL**

If the students pour all of the water into a single container, what is the total volume of water in the container?

Solution

ANALYZE

What is given in the problem? **three measured volumes of water**

What are you asked to find? **the total volume of water**

Items	Data
First volume of water	164 mL
Second volume of water	39.7 mL
Third volume of water	18.16 mL
Total volume of water	?

PLAN

What step is needed to calculate the total volume of the water?

Add the separate volumes.

What steps are necessary to round the calculated value to the correct number of significant figures?

Determine which measurement has the fewest decimal places. Round the calculated result to that number of decimal places.

COMPUTE

$$V_{total} = V_1 + V_2 + V_3 = 164\ \text{mL} + 39.7\ \text{mL} + 18.16\ \text{mL}$$

$$\begin{array}{r} 164\ \text{mL} \\ +\ 39.7\ \text{mL} \\ +\ 18.16\ \text{mL} \\ \hline 221.86\ \text{mL} \end{array}$$

Round the sum to the same number of decimal places as the measurement with the fewest decimal places (164 mL).

$$V_{total} = 221.86\ \text{mL} = 222\ \text{mL}$$

the digit following the 1 is greater than 5, so the 1 is rounded up to 2

EVALUATE

Are the units correct?

Yes; the given values have units of mL.

Are the significant figures correct?

Yes; three significant figures is correct.

Is the answer reasonable?

Yes; estimating the values as 160, 40, and 20 gives a sum of 220, which is very near the answer.

Practice

1. Perform the following calculations and express the results in the correct units and number of significant figures:

a. 22.0 m + 5.28 m + 15.5 m **ans: 42.8 m**

b. 0.042 kg + 1.229 kg + 0.502 kg **ans: 1.773 kg**

c. 170 cm^2 + 3.5 cm^2 − 28 cm^2 **ans: 150 cm^2**

d. 0.003 L + 0.0048 L + 0.100 L **ans: 0.108 L**

e. 24.50 dL + 4.30 dL + 10.2 dL **ans: 39.0 dL**

f. 3200 mg + 325 mg − 688 mg **ans: 2800 mg**

g. 14 000 kg + 8000 kg + 590 kg **ans: 23 000 kg**

Additional Problems

1. Determine the number of significant figures in the following measurements:

a. 0.0120 m

b. 100.5 mL

c. 101 g

d. 350 cm^2

e. 0.97 km

f. 1000 kg

g. 180. mm

h. 0.4936 L

i. 0.020 700 s

2. Round the following quantities to the specified number of significant figures:

a. 5 487 129 m to three significant figures

b. 0.013 479 265 mL to six significant figures

c. 31 947.972 cm^2 to four significant figures

d. 192.6739 m^2 to five significant figures

e. 786.9164 cm to two significant figures

f. 389 277 600 J to six significant figures

g. 225 834.762 cm^3 to seven significant figures

3. Perform the following calculations, and express the answer in the correct units and number of significant figures.

a. 651 cm × 75 cm

b. 7.835 kg ÷ 2.5 L

c. 14.75 L ÷ 1.20 s

d. 360 cm × 51 cm × 9.07 cm

e. 5.18 m × 0.77 m × 10.22 m

f. 34.95 g ÷ 11.169 cm^3

4. Perform the following calculations, and express the answer in the correct units and number of significant figures.

a. 7.945 J + 82.3 J − 0.02 J

b. 0.0012 m − 0.000 45 m − 0.000 11 m

c. 500 g + 432 g + 2 g

d. 31.2 kPa + 0.0035 kPa − 0.147 kPa

e. 312 dL − 31.2 dL − 3.12 dL

f. 1701 kg + 50 kg + 43 kg

5. A rectangle measures 87.59 cm by 35.1 mm. Express its area with the proper number of significant figures in the specified unit:

a. in cm^2

b. in mm^2

c. in m^2

6. A box measures 900. mm by 31.5 mm by 6.3 cm. State its volume with the proper number of significant figures in the specified unit:

a. in cm^3

b. in m^3

c. in mm^3

7. A 125 mL sample of liquid has a mass of 0.16 kg. What is the density of the liquid in the following measurements?

a. kg/m^3

b. g/mL

c. kg/dm^3

8. Perform the following calculations, and express the results in the correct units and with the proper number of significant figures.

a. 13.75 mm × 10.1 mm × 0.91 mm

b. 89.4 cm^2 × 4.8 cm

c. 14.9 m^3 ÷ 3.0 m^2

d. 6.975 m × 30 m × 21.5 m

9. What is the volume of a region of space that measures 752 m × 319 m × 110 m? Give your answer in the correct unit and with the proper number of significant figures.

10. Perform the following calculations, and express the results in the correct units and with the proper number of significant figures.

a. 7.382 g + 1.21 g + 4.7923 g

b. 51.3 mg + 83 mg − 34.2 mg

c. 0.007 L − 0.0037 L + 0.012 L

d. 253.05 cm^2 + 33.9 cm^2 + 28 cm^2

e. 14.77 kg + 0.086 kg − 0.391 kg

f. 319 mL + 13.75 mL + 20. mL

11. A container measures 30.5 mm × 202 mm × 153 mm. When it is full of a liquid, it has a mass of 1.33 kg. When it is empty, it has a mass of 0.30 kg. What is the density of the liquid in kilograms per liter?

12. If 7.76 km of wire has a mass of 3.3 kg, what is the mass of the wire in g/m? What length in meters would have a mass of 1.0 g?

13. A container of plant food recommends an application rate of 52 kg/ha. If the container holds 10 kg of plant food, how many square meters will it cover (1 ha = 10 000 m^2)?

14. A chemical process produces 974 550 kJ of heat in 37.0 min. What is the rate in kilojoules per minute? What is the rate in kilojoules per second?

Problem Solving *continued*

15. A water pipe fills a container that measures 189 cm × 307 cm × 272 cm in 97 s.

a. What is the volume of the container in cubic meters?

b. What is the rate of flow in the pipe in liters per minute?

c. What is the rate of flow in cubic meters per hour?

16. Perform the following calculations, and express the results in the correct units and with the proper number of significant figures. Note, in problems with multiple steps, it is better to perform the entire calculation and then round to significant figures.

a. $(0.054\ \text{kg} + 1.33\ \text{kg}) \times 5.4\ \text{m}^2$

b. $67.35\ \text{cm}^2 \div (1.401\ \text{cm} - 0.399\ \text{cm})$

c. $4.198\ \text{kg} \times (1019\ \text{m}^2 - 40\ \text{m}^2) \div (54.2\ \text{s} \times 31.3\ \text{s})$

d. $3.14159\ \text{m} \times (4.17\ \text{m} + 2.150\ \text{m})$

e. $690\ 000\ \text{m} \div (5.022\ \text{h} - 4.31\ \text{h})$

f. $(6.23\ \text{cm} + 3.111\ \text{cm} - 0.05\ \text{cm}) \times 14.99\ \text{cm}$

Name _____ Class _____ Date _____

Skills Worksheet

Problem Solving

Scientific Notation

People who work in scientific fields often have to use very large and very small numbers. Look at some examples in the following table:

Measurement	Value
Density of air at 27°C and 1 atm pressure	0.001 61 g/cm^3
Radius of a calcium atom	0.000 000 000 197 m
One light-year	9 460 000 000 000 km
The mass of a neutron	0.000 000 000 000 000 000 000 001 675 g

You can see that measurements such as these would be awkward to write out repeatedly. Also, calculating with very long numbers is likely to lead to errors because it's so easy to miscount zeros and decimal places. To make these numbers easier to handle, scientists express them in a form known as *scientific notation*, which uses powers of 10 to reduce the number of zeros to a minimum.

Look at a simple example of the way that scientific notation works. Following are some powers of 10 and their decimal equivalents.

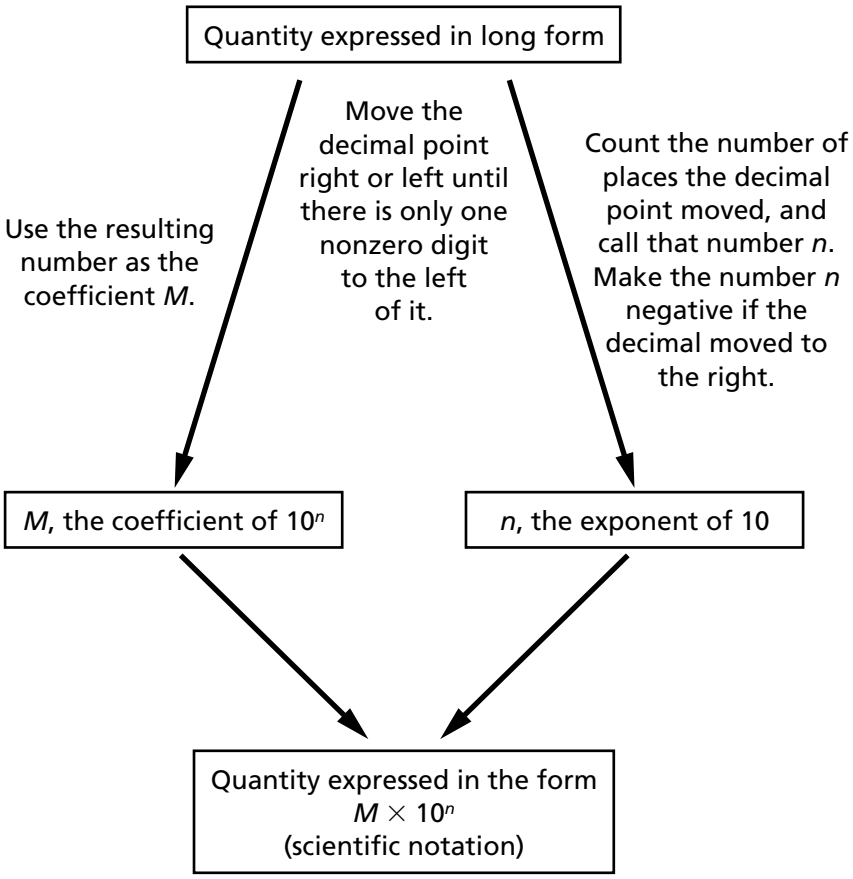
$$10^{-2} = 0.01$$
$$10^{-1} = 0.1$$
$$10^{0} = 1$$
$$10^{1} = 10$$
$$10^{2} = 100$$

Suppose we rewrite the values in the table using scientific notation. The numbers become much less cumbersome.

Measurement	Value
Density of air at 27°C and 1 atm pressure	1.61×10^{-3} g/cm^3
Radius of a calcium atom	1.97×10^{-10} m
One light-year	9.46×10^{12} km
Mass of a neutron	1.675×10^{-24} g

CONVERTING QUANTITIES TO SCIENTIFIC NOTATION

General Plan for Converting Quantities to Scientific Notation



Sample Problem 1

Express the following measurements in scientific notation.

a. 310 000 L

b. 0.000 49 kg

Solution

ANALYZE

What is given in the problem? **two measured quantities**

What are you asked to find? **the measured quantities expressed in scientific notation**

Items	Data	
	a	b
Measured quantity	310 000 L	0.000 49 kg
Quantity expressed in scientific notation	? L	? kg

PLAN

What steps are needed to rewrite the quantities in scientific notation?

Move the decimal point in each value until there is only one nonzero digit to the left of it. This number becomes the coefficient, *M*.

Count the number of places the decimal was moved. If it moved to the left, the count is a positive number. If it moved to the right, the count is a negative number. Make this number, *n*, the exponent of 10.

$$Quantity\ written\ in\ long\ form = M \times 10^{n}$$

COMPUTE

a. Express 310 000 L in scientific notation

$M = 3.1$

3,1 0 0 0 0 0 — *decimal point moves 5 places to the left*, $n = +5$

$310\ 000\ \text{L} = 3.1 \times 10^{5}\ \text{L}$

b. Express 0.000 49 kg in scientific notation.

$M = 4.9$

0. 0 0 0 4 9 — *decimal point moves 4 places to the right*, $n = -4$

$0.000\ 49\ \text{kg} = 4.9 \times 10^{-4}\ \text{kg}$

EVALUATE

	a	b
Are units correct?	**Yes; the original measurement was in liters.**	**Yes; the original measurement was in kilograms.**
Is the quantity correctly expressed?	**Yes; the decimal was moved to the left five places to give a coefficient of 3.1 and an exponent of +5.**	**Yes; the decimal was moved to the right four places to give a coefficient of 4.9 and an exponent of −4.**

Practice

1. Express the following quantities in scientific notation:

a. 8 800 000 000 m **ans: 8.8×10^{9} m**

b. 0.0015 kg **ans: 1.5×10^{-3} kg**

c. 0.000 000 000 06 kg/m^3 **ans: 6×10^{-11} kg/m^3**

d. 8 002 000 Hz **ans: 8.002×10^{6} Hz**

e. 0.009 003 amp **ans: 9.003×10^{-3} amp**

f. 70 000 000 000 000 000 km **ans: 7×10^{16} km**

g. 6028 L **ans: 6.028×10^{3} L**

h. 0.2105 g **ans: 2.105×10^{-1} g**

i. 600 005 000 kJ/h **ans: $6.000\ 05 \times 10^{8}$ kJ/h**

j. 33.8 m^2 **ans: 3.38×10^{1} m^2**

CALCULATING WITH QUANTITIES IN SCIENTIFIC NOTATION

Sample Problem 2

What is the total of the measurements 3.61×10^4 mm, 5.88×10^3 mm, and 8.1×10^2 mm?

Solution

ANALYZE

What is given in the problem? **three measured quantities expressed in scientific notation**

What are you asked to find? **the sum of those quantities**

Items	Data		
Measured quantity	3.61×10^4 mm	5.88×10^3 mm	8.1×10^2 mm

PLAN

What steps are needed to add the quantities?

Convert each quantity so that each exponent is the same as that on the quantity with the largest exponent. The quantities can then be added together. Make sure the result has the correct number of significant figures.

$(P \times 10^Q) + (R \times 10^S) + (T \times 10^V) = ?$ $\qquad$ $(P + R' + T') \times 10^Q = ?$

if the exponents are different, convert the quantities so that they have the same exponent as the term with the largest exponent

add the quantities P, R′, and T′, and multiply them by the factor 10^Q

$(P \times 10^Q) + (R' \times 10^Q) + (T' \times 10^Q) = ?$

COMPUTE

$$3.61 \times 10^4 \text{ mm} + 5.88 \times 10^3 \text{ mm} + 8.1 \times 10^2 \text{ mm} = ? \text{ mm}$$

Convert the second and third quantities to multiples of 10^4.

To convert 5.88×10^3 mm:

$$5.88 \times 10^3 \text{ mm} = M \times 10^4 \text{ mm}$$

Because one was added to the exponent, the decimal point must be moved one place to the left.

$$M = 0.588$$

To convert 8.1×10^2 mm:

$$8.1 \times 10^2 \text{ mm} = M \times 10^4 \text{ mm}$$

Because two was added to the exponent, the decimal point must be moved two places to the left.

$$M = 0.081$$

Now the three quantities can be added, as follows:

$$\begin{array}{r} 3.61 \times 10^4 \text{ mm} \\ + 0.588 \times 10^4 \text{ mm} \\ \underline{+ 0.081 \times 10^4 \text{ mm}} \\ 4.275 \times 10^4 \text{ mm} \end{array}$$

To express the result in the correct number of significant figures, note that the result should only contain two decimal places.

$$4.275 \times 10^4 \text{ mm} = 4.28 \times 10^4 \text{ mm}$$

this digit is 5, so round up

EVALUATE

Are the units correct?

Yes; units of all quantities were millimeters.

Is the quantity correctly expressed in scientific notation?

Yes; there is only one number to the left of the decimal point.

Is the quantity expressed in the correct number of significant figures?

Yes; the result was rounded to give two decimal places to match the least accurate measurement.

Practice

1. Carry out the following calculations. Express the results in scientific notation and with the correct number of significant figures.

a. 4.74×10^4 km + 7.71×10^3 km + 1.05×10^3 km **ans: 5.62×10^4 km**

b. 2.75×10^{-4} m + 8.03×10^{-5} m + 2.122×10^{-3} m **ans: 2.477×10^{-3} m**

Problem Solving *continued*

c. $4.0 \times 10^{-5}\ \text{m}^3 + 6.85 \times 10^{-6}\ \text{m}^3 - 1.05 \times 10^{-5}\ \text{m}^3$ **ans: 3.6×10^{-5} m^3**

d. $3.15 \times 10^{2}\ \text{mg} + 3.15 \times 10^{3}\ \text{mg} + 3.15 \times 10^{4}\ \text{mg}$ **ans: 3.50×10^{4} mg**

e. $3.01 \times 10^{22}\ \text{atoms} + 1.19 \times 10^{23}\ \text{atoms} + 9.80 \times 10^{21}\ \text{atoms}$
ans: 1.59×10^{23} atoms

f. $6.85 \times 10^{7}\ \text{nm} + 4.0229 \times 10^{8}\ \text{nm} - 8.38 \times 10^{6}\ \text{nm}$ **ans: 4.624×10^{8} nm**

Sample Problem 3

Perform the following calculation, and express the result in scientific notation:

$$3.03 \times 10^4\ \text{cm}^2 \times 6.29 \times 10^2\ \text{cm}$$

Solution

ANALYZE

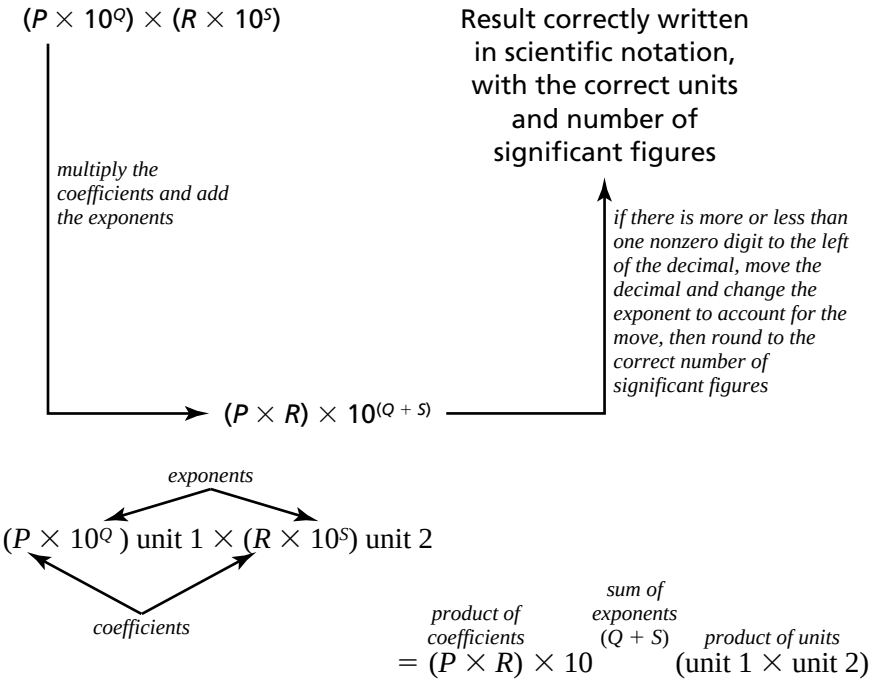
What is given in the problem? **two quantities expressed in scientific notation**

What are you asked to find? **the product of the two quantities**

Items	Data	
Measured quantity	$3.03 \times 10^4\ \text{cm}^2$	$6.29 \times 10^2\ \text{cm}$

PLAN

What steps are needed to multiply quantities expressed in scientific notation?
Multiply the coefficients, and add the exponents. Then transform to the correct scientific notation form with the correct units and number of significant figures.



COMPUTE

$3.03 \times 10^4\ \text{cm}^2 \times 6.29 \times 10^2\ \text{cm} =$

$$(3.03 \times 6.29) \times 10^{(4+2)}\ (\text{cm}^2 \times \text{cm}) = 19.0587 \times 10^6\ \text{cm}^3$$

To transform the result to the correct form for scientific notation, move the decimal point left one place and increase the exponent by one.

$$19.0587 \times 10^6 \text{ cm}^3 = 1.90587 \times 10^{(6+1)} \text{ cm}^3 = 1.90587 \times 10^7 \text{ cm}^3$$

To express the result to the correct number of significant figures, note that both of the original quantities have three significant figures. Therefore, round off the result to three significant figures.

$$1.905\,87 \times 10^7 \text{ cm}^3 = 1.91 \times 10^7 \text{ cm}^3$$

this digit is 5, so round up

EVALUATE

Are the units correct?

Yes; the units cm^2 and cm are multiplied to give cm^3.

Is the quantity expressed to the correct number of significant figures?

Yes; the number of significant figures is correct because the data were given to three significant figures.

Is the quantity expressed correctly in scientific notation?

Yes; moving the decimal point decreases the coefficient by a factor of 10, so the exponent increases by one to compensate.

Practice

1. Carry out the following computations, and express the result in scientific notation:

a. $7.20 \times 10^3 \text{ cm} \times 8.08 \times 10^3 \text{ cm}$ **ans: $5.82 \times 10^7 \text{ cm}^2$**

b. $3.7 \times 10^4 \text{ mm} \times 6.6 \times 10^4 \text{ mm} \times 9.89 \times 10^3 \text{ mm}$ **ans: $2.4 \times 10^{13} \text{ mm}^3$**

c. $8.27 \times 10^2 \text{ m} \times 2.5 \times 10^{-3} \text{ m} \times 3.00 \times 10^{-4} \text{ m}$ **ans: $6.2 \times 10^{-4} \text{ m}^3$**

d. $4.44 \times 10^{-35} \text{ m} \times 5.55 \times 10^{19} \text{ m} \times 7.69 \times 10^{-12} \text{ kg}$ **ans: $1.89 \times 10^{-26} \text{ kg} \cdot \text{m}^2$**

e. $6.55 \times 10^4 \text{ dm} \times 7.89 \times 10^9 \text{ dm} \times 4.01893 \times 10^5 \text{ dm}$ **ans: $2.08 \times 10^{20} \text{ dm}^3$**

Sample Problem 4

Perform the following calculation, and express the result in scientific notation:

$$3.803 \times 10^3 \text{ g} \div 5.3 \times 10^6 \text{ mL}$$

Solution

ANALYZE

What is given in the problem? **two quantities expressed in scientific notation**

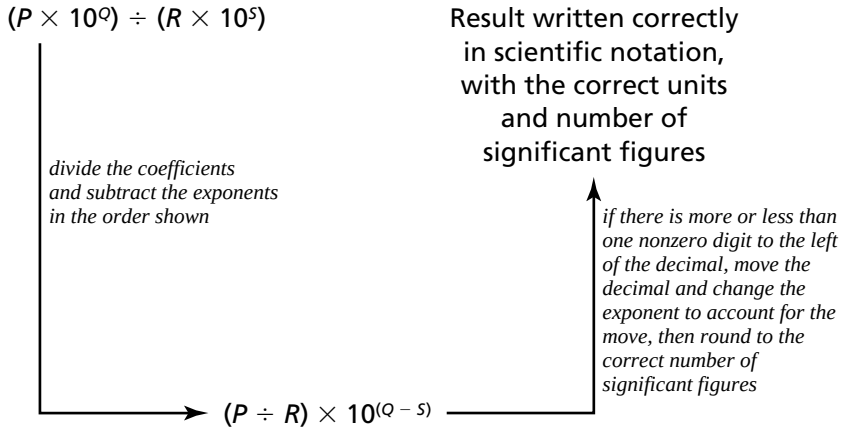
What are you asked to find? **the quotient of the two quantities**

Items	Data	
Measured quantity	3.803×10^3 g	5.3×10^6 mL

PLAN

What steps are needed to divide the quantities expressed in scientific notation?

Divide the coefficients, and subtract the exponents. Then transform the result to the correct form for scientific notation with the correct units and number of significant figures.



$$\frac{P \times 10^Q \text{ unit 1}}{R \times 10^S \text{ unit 2}} = \underbrace{\frac{P}{R}}_{\text{quotient of coefficients}} \times \underbrace{10^{(Q-S)}}_{\text{difference of exponents}} \underbrace{\frac{\text{unit 1}}{\text{unit 2}}}_{\text{quotient of units}}$$

COMPUTE

$$3.803 \times 10^3 \text{ g} \div 5.3 \times 10^6 \text{ mL} = \frac{3.803}{5.3} \times 10^{(3-6)} \frac{\text{g}}{\text{mL}} = 0.717\,547 \times 10^{-3} \text{ g/mL}$$

The measurement 5.3×10^6 mL has the fewest significant figures; round the result accordingly.

$$0.717\,\cancel{547} \times 10^{-3} \text{ g/mL} = 0.72 \times 10^{-3} \text{ g/mL}$$

this digit is greater than 5, so round up

To transform the result to the correct form for scientific notation, move the decimal point to the right one place and decrease the exponent by one.

$$0.72 \times 10^{-3}\ \text{g/mL} = 7.2 \times 10^{(-3-1)}\ \text{g/mL} = 7.2 \times 10^{-4}\ \text{g/mL}$$

EVALUATE

Are the units correct?

Yes; grams divided by milliliters gives g/mL, a unit of density.

Is the quantity expressed to the correct number of significant figures?

Yes; the result was limited to two significant figures by the data given.

Is the quantity expressed correctly in scientific notation?

Yes; there is only one nonzero digit to the left of the decimal point.

Practice

1. Carry out the following computations, and express the result in scientific notation:

a. $2.290 \times 10^{7}\ \text{cm} \div 4.33 \times 10^{3}\ \text{s}$ **ans: 5.29×10^{3} cm/s**

b. $1.788 \times 10^{-5}\ \text{L} \div 7.111 \times 10^{-3}\ \text{m}^2$ **ans: 2.514×10^{-3} L/m^2**

c. $5.515 \times 10^{4}\ \text{L} \div 6.04 \times 10^{3}\ \text{km}$ **ans: 9.13 L/km**

d. $3.29 \times 10^{-4}\ \text{km} \div 1.48 \times 10^{-2}\ \text{min}$ **ans: 2.22×10^{-2} km/min**

e. $4.73 \times 10^{-4}\ \text{g} \div (2.08 \times 10^{-3}\ \text{km} \times 5.60 \times 10^{-4}\ \text{km})$ **ans: 4.06×10^{2} g/km^2**

Additional Problems

1. Express the following quantities in scientific notation:

a. 158 000 km

b. 0.000 009 782 L

c. 837 100 000 cm^3

d. 6 500 000 000 mm^2

e. 0.005 93 g

f. 0.000 000 006 13 m

g. 12 552 000 J

h. 0.000 008 004 g/L

i. 0.010 995 kg

j. 1 050 000 000 Hz

2. Perform the following calculations, and express the result in scientific notation with the correct number of significant figures:

a. 2.48×10^2 kg + 9.17×10^3 kg + 7.2×10^1 kg

b. 4.07×10^{-5} mg + 3.966×10^{-4} mg + 7.1×10^{-2} mg

c. 1.39×10^4 m^3 + 6.52×10^2 m^3 − 4.8×10^3 m^3

d. 7.70×10^{-9} m − 3.95×10^{-8} m + 1.88×10^{-7} m

e. 1.111×10^5 J + 5.82×10^4 J + 3.01×10^6 J

f. 9.81×10^{27} molecules + 3.18×10^{25} molecules − 2.09×10^{26} molecules

g. 1.36×10^7 cm + 3.456×10^6 cm − 1.01×10^7 cm + 5.122×10^5 cm

3. Perform the following computations, and express the result in scientific notation with the correct number of significant figures:

a. 1.54×10^{-1} L ÷ 2.36×10^{-4} s

b. 3.890×10^4 mm × 4.71×10^2 mm^2

c. 9.571×10^3 kg ÷ 3.82×10^{-1} m^2

d. 8.33×10^3 km ÷ 1.97×10^2 s

e. 9.36×10^2 m × 3.82×10^3 m × 9.01×10^{-1} m

f. 6.377×10^4 J ÷ 7.35×10^{-3} s

4. Your electric company charges you for the electric energy you use, measured in kilowatt-hours (kWh). One kWh is equivalent to 3 600 000 J. Express this quantity in scientific notation.

5. The pressure in the deepest part of the ocean is 11 200 000 Pa. Express this pressure in scientific notation.

6. Convert 1.5 km to millimeters, and express the result in scientific notation.

7. Light travels at a speed of about 300 000 km/s.

a. Express this value in scientific notation.

b. Convert this value to meters per hour.

c. What distance in centimeters does light travel in 1 μs?

8. There are 7.11×10^{24} molecules in 100.0 cm^3 of a certain substance.

a. What is the number of molecules in 1.09 cm^3 of the substance?

b. What would be the number of molecules in 2.24×10^4 cm^3 of the substance?

c. What number of molecules are in 9.01×10^{-6} cm^3 of the substance?

9. The number of transistors on a particular integrated circuit is 3 578 000, and the integrated circuit measures 9.5 mm × 8.2 mm.

a. What is the area occupied by each transistor?

b. Using your answer from (a), how many transistors could be formed on a silicon sheet that measures 353 mm × 265 mm?

10. A solution has 0.0501 g of a substance in 1.00 L. Express this concentration in grams per microliter.

11. Cesium atoms are the largest of the naturally occurring elements. They have a diameter of 5.30×10^{-10} m. Calculate the number of cesium atoms that would have to be lined up to give a row of cesium atoms 2.54 cm (1 in.) long.

12. The neutron has a volume of approximately 1.4×10^{-44} m^3 and a mass of 1.675×10^{-24} g. Calculate the density of the neutron in g/m^3. What is the mass of 1.0 cm^3 of neutrons in kilograms?

13. The pits in a compact disc are some of the smallest things ever mass-produced mechanically by humans. These pits represent the *1*s and *0*s of digital information on a compact disc. These pits are only 1.6×10^{-8} m deep (1/4 the wavelength of red laser light). How many of these pits would have to be stacked on top of each other to make a hole 0.305 m deep?

14. 22 400 mL of oxygen gas contains 6.022×10^{23} oxygen molecules at 0°C and standard atmospheric pressure.

a. How many oxygen molecules are in 0.100 mL of gas?

b. How many oxygen molecules are in 1.00 L of gas?

c. What is the average space in milliliters occupied by one oxygen molecule?

15. The mass of the atmosphere is calculated to be 5.136×10^{18} kg, and there are 6 500 000 000 people living on Earth. Calculate the following values.

a. The mass of atmosphere in kilograms per person.

b. The mass of atmosphere in metric tons per person.

c. If the number of people increases to 9 500 000 000, what is the mass in kilograms per person?

16. The mass of the sun is 1.989×10^{30} kg, and the mass of Earth is 5.974×10^{24} kilograms. How many Earths would be needed to equal the mass of the sun?

Problem Solving *continued*

17. A new landfill has dimensions of 2.3 km × 1.4 km × 0.15 km.

a. What is the volume in cubic kilometer?

b. What is the volume in cubic meters?

c. If 250 000 000 objects averaging 0.060 m^3 each are placed into the landfill each year, how many years will it take to fill the landfill?

18. A dietary calorie (C) is exactly equal to 1000 cal. If your daily intake of food gives you 2400 C, what is your intake in joules per day? (1 cal = 4.184 J)